

A Competition Model for Battle Games

Bruno Larentis
Marta Becker Villamil
Luiz Paulo Luna de Oliveira

Programa Interdisciplinar de Pós-Graduação em Computação Aplicada
Universidade do Vale do Rio dos Sinos

Abstract

Games with a competition factor between two or more groups need a model to manage the many variables involved in a system where there is a correlation between the groups. Usually this kind of game leaves the strategic decisions to the player and use them as parameters of the simulation model. This paper proposes the use of the Lotka-Volterra model for simulating competitive groups. These ordinary differential equations are adapted to best fit the context of battle games. To illustrate the adaptation of the competition model, we suggest a game consisting of a battle situation where the local citizens of a dominated city must compete with invaders for getting a hidden important medicine. We analyze the model with respect to the reinforcement parameter via standard stability analysis.

Keywords:: Real-time Strategy, Battle Games, Interacting Populations

Author's Contact:

survivorhunk@yahoo.com.br, {mbvillamil, lpluna}@unisinos.br

1 Introduction

When people become interrelated, as when they are members of a group, they develop patterned relationships among themselves—patterned in terms of status, of power, of affection, and many other aspects. One could say that they become one single entity with the same motivation to perform a task.

In the case of military strategy the task concerns to deal with the planning and conduct campaigns, the movement and disposition of forces, and the deception of the enemy. Military swarming involves the use of a decentralized force against an opponent, in a manner that emphasizes mobility, communication, unit autonomy and coordination/synchronization. It is often said that the art of strategies defines the goals to achieve in a military campaign, while tactics defines the methods to achieve these goals.

War games give the player god-like control over the battle. The entertainment comes from observing the effects of player actions over the warriors and their groups. The level of strategies for achieve the goals has to be better than the group of enemies. Typically, other factors such as positioning in enemy territory, the surprise attack and the number of warriors of the armies involved also affect substantially the evolution of the battle in favor of either army. Some variables evolved in such games (knowledge of territories, kind and quantity of weapons, army positioning, number of warriors...) depends on the situation that the player are conducting and training his army but also depends of the conditions of the other army group. There is an interconnection between the resources of the armies and the result of battle.

During the battles, usually, the ratio of resources between the armies is inversely proportional. Variables as number of warriors and quantity of weapons can be affected by the other army in a battle. In most cases, if an army is with better resources, it means that it will win the battle and reduce the resources of the opposing army. Unless the army which is at disadvantage has to compensate this loss with a good strategy. If variables are similar for both armies including strategies and number of warriors, the battle will persist until one army has a lower or higher level in some variable to destabilize the system.

This work proposes a model to be applied to a battle game with two armies. The competition factor, strategies and number of warriors are modeled using Lotka-Volterra [Lotka 1920; Volterra 1926] differential equations in order to deal with oscillatory levels of variables between the two groups.

The organization of this paper is described as follows: next section describes briefly some related work that deal with battle games and interaction of groups of agents. Section 3 describes Lotka-Volterra model for competitive groups. Section 4 describes the game where the groups compete in a battle managed by the population dynamics equations. Section 6 draws some conclusions and future extensions of this work.

2 Related Work

Most of simulation games has groups that interact with significant autonomy. Group interaction involves the group *doing something*. The groups have their own drives, goals, and strategies for satisfying those goals, but the human player can come in and stir things up both by managing the groups and individual characters and the environment. Any groups interaction in games can be characterized in terms of the task(s) that the group is trying to carry out: playing soccer [Konami 2010; EA 2010a], playing basketball [EA 2010b], walking, dancing, taking dinner [ElectronicArts 2010], fighting [Activision 2010; Ubisoft 2010].

At Age of Empires [Microsoft 2010], a well-known strategy game, the group controlled by the player is set on a certain region and has to explore it to get new resources. At any time, the player can upgrade his group as well as their conditions. Those resources are upgraded to turn the player able to have more areas under and to survive from attacks of other groups. This kind of battle game is decided if the player is able to administrate well his resources to avoid the extermination of his group.

Other game that have the same gameplay system is Rome: Total War [SEGA 2010]. However, instead of been focused on the evolution of the group, tools, constructions, this game is focused on conquering regions by war. With this goal in mind, the player needs to have two basic abilities: one is to know how to upgrade his army, other is to conquer new territories, and other is to survive from attacks made by other groups.

On both games the players resources keep suffering variations. Those variations occur in good and bad ways; if the player loses a battle her/his resources suffer a reduction and if the player don't work to get more resources her/his chances of losing the game are higher. But if he wins a battle or finds new regions he can accumulate more resources and the chance of wining goes higher.

Concerning the study of people group, [Jacques et al. 2007] analyzed individual parameters as perceived personal space and comfort levels in video sequences using Voronoi diagrams to detect formation of groups and their classification as voluntary or involuntary. [Champagne and Tang 2005] also use Voronoi diagrams to simulate crowds. In this case each group of agents has its own Voronoi region. During the simulation, agents in groups move closely together inside the region and avoid collision with other groups on the way.

[Villamil et al. 2005] proposed a simulation based on the interaction of groups with competitive behavior also based on Lotka-Volterra population model. The goal of the groups is to co-opt new members from the opposing group. Model parameters are degree of compe-

tion and cohesion of groups. For forming groups, individual have the ability of perceiving others and interacting. Thus, they form groups with individuals who they have had good relationship based on certain parameters such as sociability, communication and comfort.

3 Continuous Models for Interacting Populations

When species interact, the population dynamics of each one is affected. *Volterra* [Volterra 1926] firstly proposed a simple model for the predation of one specie by another one in order to explain the oscillatory levels of certain fishes in the Adriatic sea. The model is represented by:

$$\frac{dN}{dt} = aN - bNP = N(a - bP), \quad (1)$$

$$\frac{dP}{dt} = cNP - dP = P(cN - d), \quad (2)$$

where N and P represent the prey and predator numbers, respectively.

According to those equations:

- In the absence of any predation, the prey grows unboundedly as indicated by aN term, where a is the it's linear birth rate.
- The effect of the predation is to reduce the prey's per capita growth rate by a term proportional to the prey and predator populations; this is $-bNP$ term.
- In the absence of any prey for sustenance, the predator's death results in exponential decay. That is the $-dP$ term, where d is the predator linear death rate.
- The prey's contribution to the predator's growth rate is cNP ; that is, it is proportional to the available prey as well as to the size of the predator population. The cNP term can be thought of as representing the conversion of energy from one source to another: bNP is taken from the prey and cNP accrues to the predators.

That model is known as the *Lotka-Volterra* model since same equations were also derived by *Lotka* [Lotka 1920] from a hypothetical chemical reaction which he said could exhibit periodic behavioral in the chemical concentrations.

The competitive Lotka-Volterra model is given by ordinary differential equations that gives the number of individuals of two competing species. Without the presence of the other, each specie develops according to a linear growth and a quadratic death rates where, the last depend on the carrying capacities of the environment for the corresponding specie. The influence of a specie on the other is given by a cross product term which represents the competition effect of one specie on the other. More precisely, the competitive Lotka-Volterra equations are

$$\frac{dN_1}{dt} = r_1 N_1 \left[1 - \frac{N_1}{K_1} - b_{12} \frac{N_2}{K_1} \right], \quad (3)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left[1 - \frac{N_2}{K_2} - b_{21} \frac{N_1}{K_2} \right], \quad (4)$$

where:

- N_1 and N_2 represent the number of individuals of group 1 (G_1) and group 2 (G_2) respectively;
- r_1 and r_2 are the respective linear growing rates;
- K_1 and K_2 are the carrying capacities of the environment for group 1 and 2 respectively;
- The b_{12} and b_{21} are the measure of competitive effect of N_2 on N_1 and N_1 on N_2 respectively;

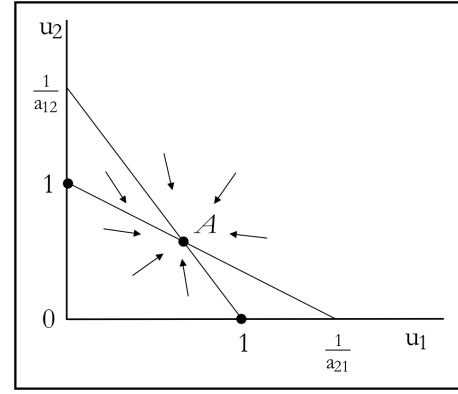


Figure 1: Case A, $a_{12} < 1$, $a_{21} < 1$. Only the positive steady state A is stable and all trajectories tend to it.

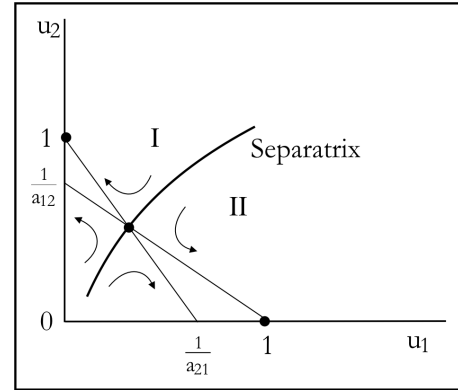


Figure 2: Case B, $a_{12} > 1$, $a_{21} > 1$. Here $(1,0)$ and $(0,1)$ are stable steady states each of which has a domain of attraction separated by a separatrix.

In order to facilitate the analysis, the above equations can be normalized by the following rules [Murray 1989]:

$$u_1 = \frac{N_1}{K_1}, \quad (5)$$

$$u_2 = \frac{N_2}{K_2}, \quad (6)$$

$$a_{12} = b_{12} \frac{K_2}{K_1}, \quad (7)$$

$$a_{21} = b_{21} \frac{K_1}{K_2}, \quad (8)$$

$$\rho = \frac{r_2}{r_1}, \quad (9)$$

$$\tau = r_1 t. \quad (10)$$

With this normalization, the original Lotka-Volterra equations (3) and (4) becomes non dimensional and reads

$$\frac{du_1}{d\tau} = u_1 (1 - u_1 - a_{12} u_2), \quad (11)$$

$$\frac{du_2}{d\tau} = \rho u_2 (1 - u_2 - a_{21} u_1), \quad (12)$$

whose stationary solutions are obtained solving the algebraic equations $\frac{du_1}{d\tau} = 0$ and $\frac{du_2}{d\tau} = 0$.

The analytical study of equations (11) and (12), gives the following four asymptotic possibilities, depicted in Figs. 1-4 [Murray 1989; Jiang and Shi 2009].

Case A: If $a_{12} < 1$ and $a_{21} < 1$ there exists a stable equilibrium point A , which means that both species will coexist and their states tend to respective coordinates of A . We say that the competition is not aggressive (Fig. 1).

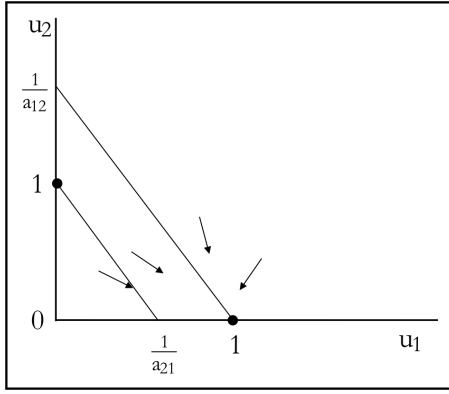


Figure 3: Case C, $a_{12} < 1$, $a_{21} > 1$. Only one stable steady state exists.

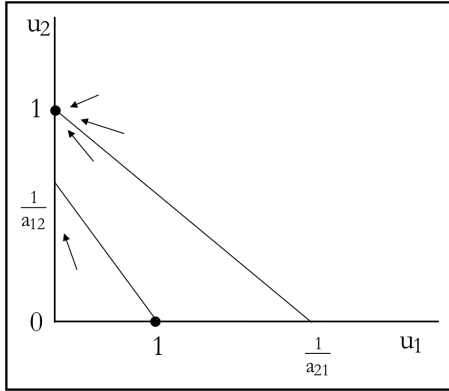


Figure 4: Case D, $a_{12} > 1$, $a_{21} < 1$. Only one stable steady state exists.

Case B: If $a_{12} > 1$ and $a_{21} > 1$, stable points $(1,0)$ and $(0,1)$ coexist with repeller $(0,0)$ and saddle point (u_1^*, u_2^*) , $u_1^* = (1 - a_{12})/(1 - a_{12}a_{21})$ and $u_2^* = (1 - a_{21})/(1 - a_{12}a_{21})$. Therefore, the system is bistable since it possesses two locally stable equilibrium points $(1,0)$ and $(0,1)$ and a separatrix (the stable manifold of (u_1^*, u_2^*)). In this case, only one specie survives, depending on the initial state of the system. Indeed, if the initial state $(u_1(0), u_2(0))$ is below the depicted separatrix, the specie 2 extinctions while, if the initial state is above the separatrix, specie 1 extinctions. In this case, we say that the competition is aggressive (Fig. 2).

Case C: If $a_{12} < 1$ and $a_{21} > 1$, the competition goes toward the extinction of species 2, independently of the initial state $(u_1(0), u_2(0))$. Then, it is also an aggressive competition scenario (Fig. 3).

Case D: If $a_{12} > 1$ and $a_{21} < 1$, the opposite competition goes toward the extinction of species 1, also independently of the initial state $(u_1(0), u_2(0))$ (Fig. 4).

4 Applying the Differential Equation Model in a Game

4.1 Interacting Battle Groups

We adapted Lotka-Volterra equations to define the rules of confrontation between two groups in the context of the proposed game,

$$\frac{dx}{dt} = v_1 + a(y - x) + by(x - y), \quad (13)$$

$$\frac{dy}{dt} = v_2 + c(x - y) + dx(y - x), \quad (14)$$

where

- the variable x represents the number of members of the group 1,
- the variable y represents the number of members of the group 2,
- the parameter v_1 represents eventual reinforcement of group 1,
- the parameter v_2 represents eventual reinforcement of group 2,
- the parameter a represents the efficiency of group 1 strategy,
- the parameter b represents the weapon power of group 1,
- the parameter c represents the efficiency of group 2 strategy,
- the parameter d represents the weapon power of group 2.

By hypothesis, only nonnegative values are accepted for the parameters.

These equations governs the dynamics of the groups while the battle takes place. In other words, the number of individuals x and y in group 1 and 2, respectively, varies according with those equations. Besides eventual apport of new elements, given by v_1 and v_2 , the factors taken into consideration in (13) and (14) are the following. The efficiency of the strategies applied by each group will raise the reaction to the disadvantaged situations, in both equations. This effect is represented by the terms $a(y - x)$ and $c(x - y)$. The confrontation between both groups is represented by the terms $by(x - y)$ and $dx(y - x)$, where the parameters corresponds to the weapon power of each group. These terms are somewhat analogous to the nonlinear ones on the mentioned competitive Lotka-Volterra model. It is proportional to the difference between the groups contingents and to the number of elements of each group.

The dynamic of the battle then results from confrontation taking into consideration the applied strategies and the weapon power of each group, as well as eventual reinforcements.

4.2 Game Rules and Parameters

This kind of model can be applied to a game based on a story of a group of survivors looking for a scientist in a dominated city by an enemy. They need to get a medication for a sick child from this scientist. Suggestion of game rules, interfaces and parameters will be explained next.

Player leads a group of people and can change its position. To find the scientist, the player have to look at each building of the city and hope that he is still alive. There is a city map that appears in an auxiliary windows of the game interface to help the player localize himself. Figure 5 shows the representation of the city and map at figure corner right. The location of the group is represented by a red building in the map. Each time the player wants that his group moves, s/he has to choose another location in the map. So, the group moves to another place. Every move costs some energy, and to go to some locations cost more energy than others. The cost of energy is calculated based on the distance between the building where the player is and the building that s/he wants to go.

Every time the player choose to move his group, some events can happen. The first one is to loose energy. The other is to be attacked by a group of enemies that are dominating the city Figure ???. The enemies attack occurs only outdoor. Inside the buildings the group is saved. If this event occurs, the camera is positioned with its focus at both groups, at one side are the survivors and on the other side are the enemies that are attacking.

Battle happens in turns. At each turn the player is able to choose his battle options from a menu list. The actions are: how many people of his group will be sent for attack, as well their weapons and positions on the battle map. Each of those actions are represented by a value that is a parameter of the equations explained at the previous section.

Number of soldiers sent to the attack is the parameter v_1 at the equation 13, the weapons selected is represent the parameter b and the

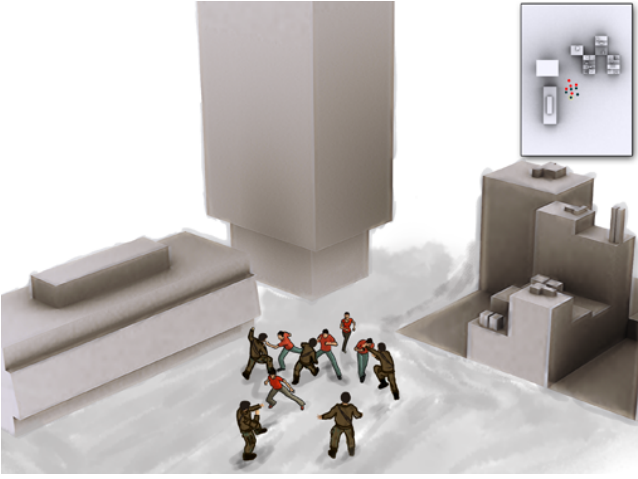


Figure 5: Virtual city where the game occurs and its map. Red shirt characters belong to group 1 and are controlled by the Player.

location on the map represents the strategy and it is the parameter a at same equation. The way that the computer find randomly the enemies on the battle, as well as the number of them and weapons is determinate on the equations too. In this case equation 14. But, at this time the parameters are v_2 (representing the number of enemies), c (representing their weapons) and d (representing their position at the battle map).

During a battle, the player can lose a few members of his group depending on the equations parameters of his group and the enemies group. The game ends if one group loses all its members. If the player win a turn he is able to send again the same members for the next turn. All the survivors of the group, as they keep going on battles, they can get some bonus points represented by weapons. This bonus represents a higher value to the b at the equation 13.

Starting the game, the group of the player receives standard parameters (number of members, weapons and energy points), after the battle ends and the player arrives at the chosen location, it receives his reward. The reward will be determined with some relation to the location that the player chooses to go. For example, if he chooses to go to the police station, his chances to find more weapons is higher, as well as if he chooses to go to a hospital he have more chances to find energy points.

If the player runs out of energy points, or he ends up at one location that he does not have enough energy points to move himself, the player is given the option to rest. When the player chooses this option he will gain a number of energy points and will remain at his location. After the player rest, there will be a chance that he will suffer a surprise attack even if the group is indoor a building, this surprise attack will be a battle. Then the interface will change for the same one as the standard battle. Different from the standard battle, that gives no reward, the reward is given when the player reaches the chosen location, this battle will give a reward that can be any of the previous rewards listed. The game will keep going until the player find the scientist and the survivors can save the children.

5 Discussion

Equations (13) and (14) give rise to a variety of scenarios, depending on the configurations of the available parameters. As an illustration here we consider some simple cases. First we consider the scenario where all the parameters of both equations coincide. In this case, the model (13) and (14) reduces to

$$\frac{dx}{dt} = v_1 + a(y - x) + by(x - y), \quad (15)$$

$$\frac{dy}{dt} = v_1 + a(x - y) + bx(y - x). \quad (16)$$

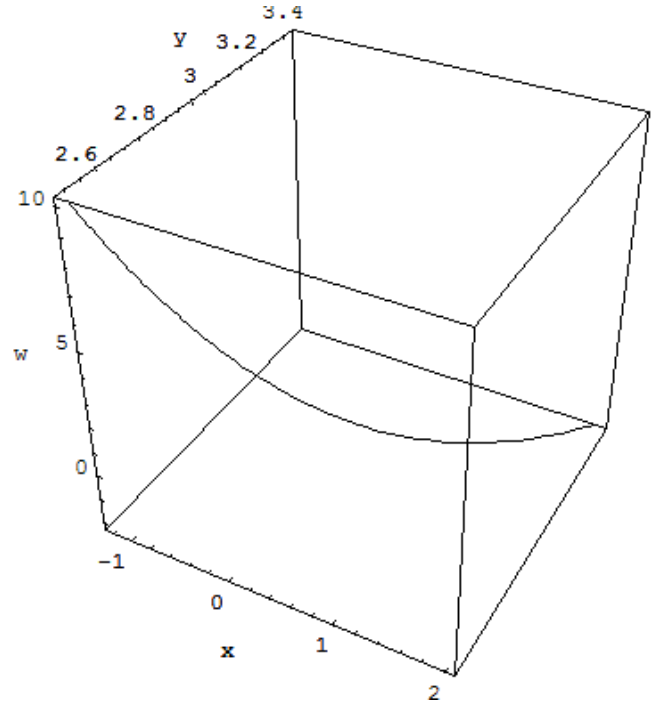


Figure 6: Position of singularity P_1 as a function of w (see Equations (19) and (20)).

Making both dx/dt and dy/dt equal to zero, we found the singular points

$$P_1 = \left(\frac{2a - \sqrt{2bv_1}}{2b}, \frac{2a + \sqrt{2bv_1}}{2b} \right) \quad (17)$$

$$P_2 = \left(\frac{2a + \sqrt{2bv_1}}{2b}, \frac{2a - \sqrt{2bv_1}}{2b} \right), \quad (18)$$

with $\lambda = \pm 2i\sqrt{bv_1}$ as eigenvalues for both P_1 and P_2 . This indicates a periodic evolution, for any initial condition (x_0, y_0) . This is what we can see from Figure 10.

The other scenario we consider here is the one for which all the parameters for (13) and (14) are respectively equal, except v_1 and v_2 . This corresponds to a situation where both groups have the same weapon power and equally effective strategies. Only the reinforcements of new members are allowed to be different. Under this assumption, the model (13) and (14) reduces to

$$\frac{dx}{dt} = v_1 + a(y - x) + by(x - y), \quad (19)$$

$$\frac{dy}{dt} = (v_1 + w) + a(x - y) + bx(y - x), \quad (20)$$

where w represents the difference between both reinforcements. In this case, an analytical study is very complicated. Therefore, here, we make an numerical study, adopting the values $a = 2$, $b = 1$; and $v_1 = 2$, and making w to vary between $w = -2$ and $w = 10$. For these cases, the system still has two singularities P_1 and P_2 , however with a much more complicated expression.

In Figures 6 and 7 we see the dependence of P_1 with respect to w , as well as the corresponding eigenvalues. Analogously, Figures 8 and 9 shows the same for singularity P_2 . We can see that, as w raises from $w = 0$, P_1 goes towards a direction where group 1 extinguishes, still maintaining its attractive nature (stability). On the other hand, for the same variation of w , P_2 gets progressively apart from the origin, while maintaining its repulsive nature (instability). This gives a global picture of an asymptotic dynamics towards P_1 . In the complementary range of variation, i.e., for $2 < w < 0$, we have the contrary behavior.

Figure 10 shows the behavior of the equations with all attributes with the same values: a is equal to c , b is equal to d and v_1 is

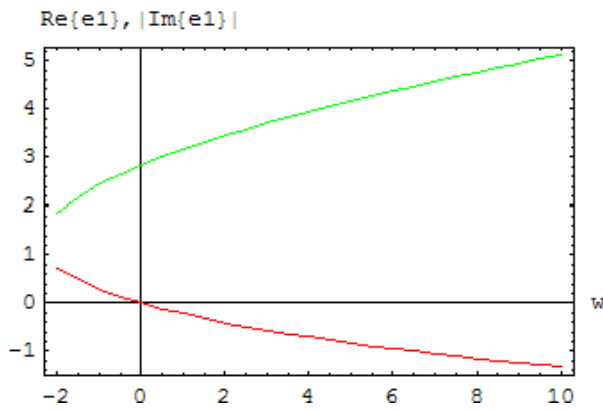


Figure 7: Real (red) and absolute imaginary (green) parts of eigenvalues associated to the singularity P_1 , as a function of w .

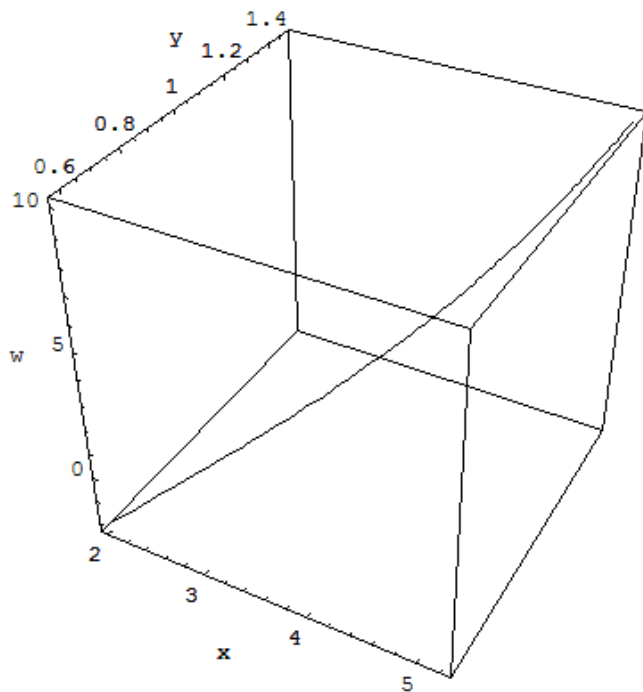


Figure 8: Position of singularity P_2 as a function of w (see Equations (19) and (20)).

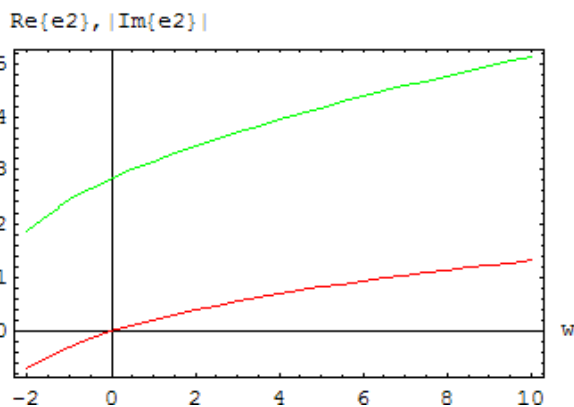


Figure 9: Real (red) and absolute imaginary (green) parts of eigenvalues associated to the singularity P_1 , as a function of w .

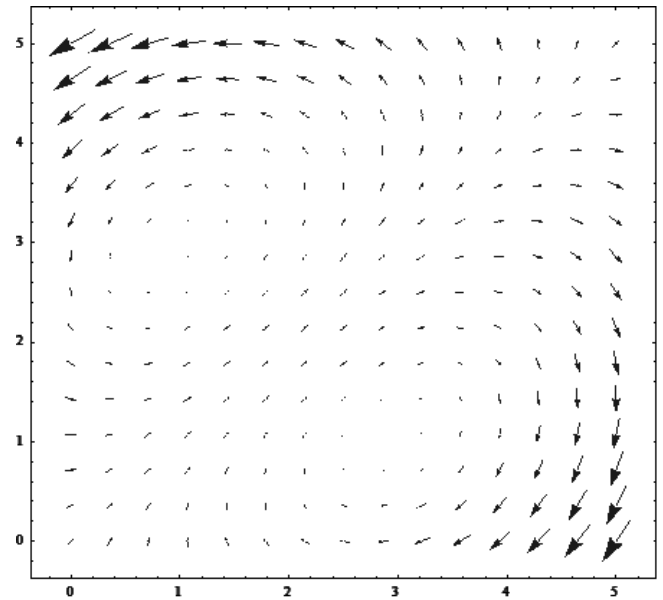


Figure 10: Dynamical behavior of groups 1 and 2 with the same parameters. See equations (15) and (16).

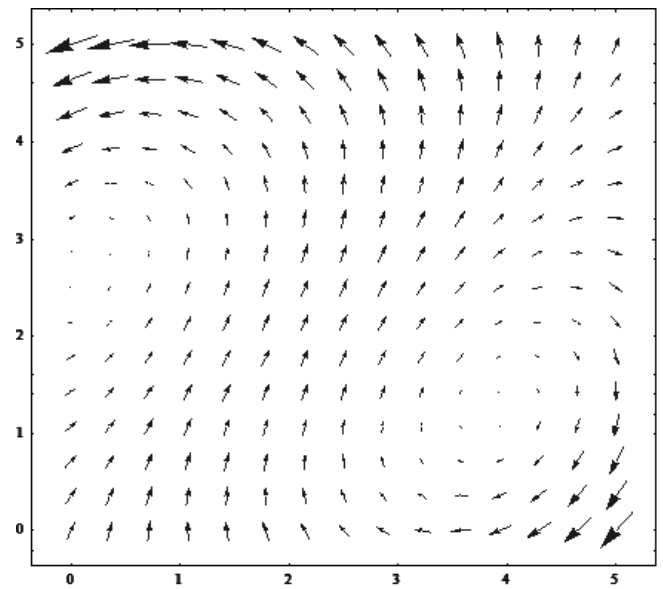


Figure 11: Dynamical behavior of both groups with v_2 value greater than v_1 .

equal to v_2 . This chart shows that the groups make variations on two similar areas of phase space. Those areas are ellipse shaped areas. This behavior indicates that with those equations and all same attributes (parameters) the number of members of each group will keep changing, in a oscillatory fashion. More precisely, group 1 (x) has more members until the group 2 (y) sufficiently increases, causing group 1 to low its members. After a while, when group y raises sufficiently, the same occur in a reverse direction, i.e. with group x increasing.

Figure 11 shows another changing made at the equations values, by leaving v_1 with a fixed value and v_2 value greater than v_1 . This chart represents the behavior created by an increased value on v_2 upon v_1 . Observing on the direction of the arrows its noticeable that the group y will overcome group x no matter the initial state if group y . If group y starts with more members than group x its very clear that group x will be exterminated very quickly. In other case, group y with less members will be close to be exterminated, but since his v_2 values is higher, they will get more members from time to time, leaving also to the group x extermination, but will take more time to this objective be reached.

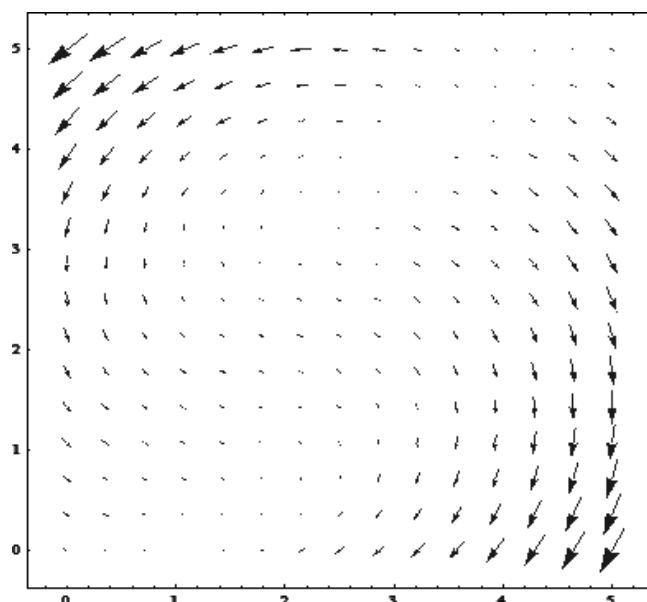


Figure 12: Dynamical behavior of both groups with $v1$ value greater than $v2$.

Figure 12 shows the same previous variation, but now with $v1$ value higher than $v2$. As before, occurs the same behavior, but now with group x overcoming group y . Again the initial state will not change the result of this battle, when one of the groups have more new members from time to time more is the chance of exterminating the other group.

6 Conclusion and Future Work

This article proposes a model to simulate the competition factor between two groups in a battle game. The Lotka-Volterra equations used to simulate animals population interaction were adapted to best fit to the game context. It has been also presented a game suggestion where this kind of model can be applied. Others games like battle between countries, soccer supports, political parties and others can be benefited with a model like this by changing some parameters and adapting them to the context of the game. In the future, we intend to implement the proposed game scenario and also improve the model to support more groups (not, only two), and let them emerge in a period of time depending on the game purpose. We also can improve the model to put together others factors like union of groups and collaboration and use some formal framework/notation to specify rules and parameters.

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